# Bayes' Theorem

# Bayes' Theorem

Helps us find the probability of an event based on partial information.

Used extensively in machine learning and other areas. Spam filters, classifying text, images, etc.

Suppose we have two jars of skittles, Jar A and Jar B. We can't see inside and don't know which is which.

Jar A contains two green skittles and seven red skittles. Jar B contains four green skittles and three red skittles.

You randomly choose a skittle from one of the jars. If the skittle is red, what is the probability you selected from Jar A?

# Bayes' Theorem

*Bayes' Theorem:*

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P\left(B\middle|\overline{A}\right) \cdot P\left(\overline{A}\right)}$$

Jar A: 2 green, 7 red
Jar B: 4 green, 3 red

You randomly choose a skittle from one of the jars. If the skittle is red, what is the probability you selected from Jar A?

$R = \{\text{Ar1}, \text{Ar2}, \text{Ar3}, \text{Ar4}, \text{Ar5}, \text{Ar6}, \text{Ar7}, \text{Br1}, \text{Br2}, \text{Br3}\}, \quad \overline{R} = \{\text{Ag1}, \text{Ag2}, \text{Bg1}, \text{Bg2}, \text{Bg3}, \text{Bg4}\}$

$A = \{\text{Jar A}\} \quad \overline{A} = \{\text{Jar B}\}$

We are looking for $P(A \mid R)$, the probability we chose from Jar A given a red skittle.

We know the following:

$P(R|A) = \frac{7}{9}$

$P\left(R\middle|\overline{A}\right) = \frac{3}{7}$

$P(A) = P\left(\overline{A}\right) = \frac{1}{2}$

*Apply Bayes' Theorem:*

$$P(A|R) = \frac{\frac{7}{9} \cdot \frac{1}{2}}{\frac{7}{9} \cdot \frac{1}{2} + \frac{3}{7} \cdot \frac{1}{2}} = \frac{\frac{7}{9}}{\frac{7}{9} + \frac{3}{7}} \approx 0.645$$

Before choosing a red skittle, we assumed the probability that we selected Jar A was 50%. But with the extra information of choosing a red skittle, that probability increased to 64.5%.

Rosen, Kenneth. *Discrete Mathematics and Its Applications, 7th Edition*. McGraw-Hill, 2012

Try it:

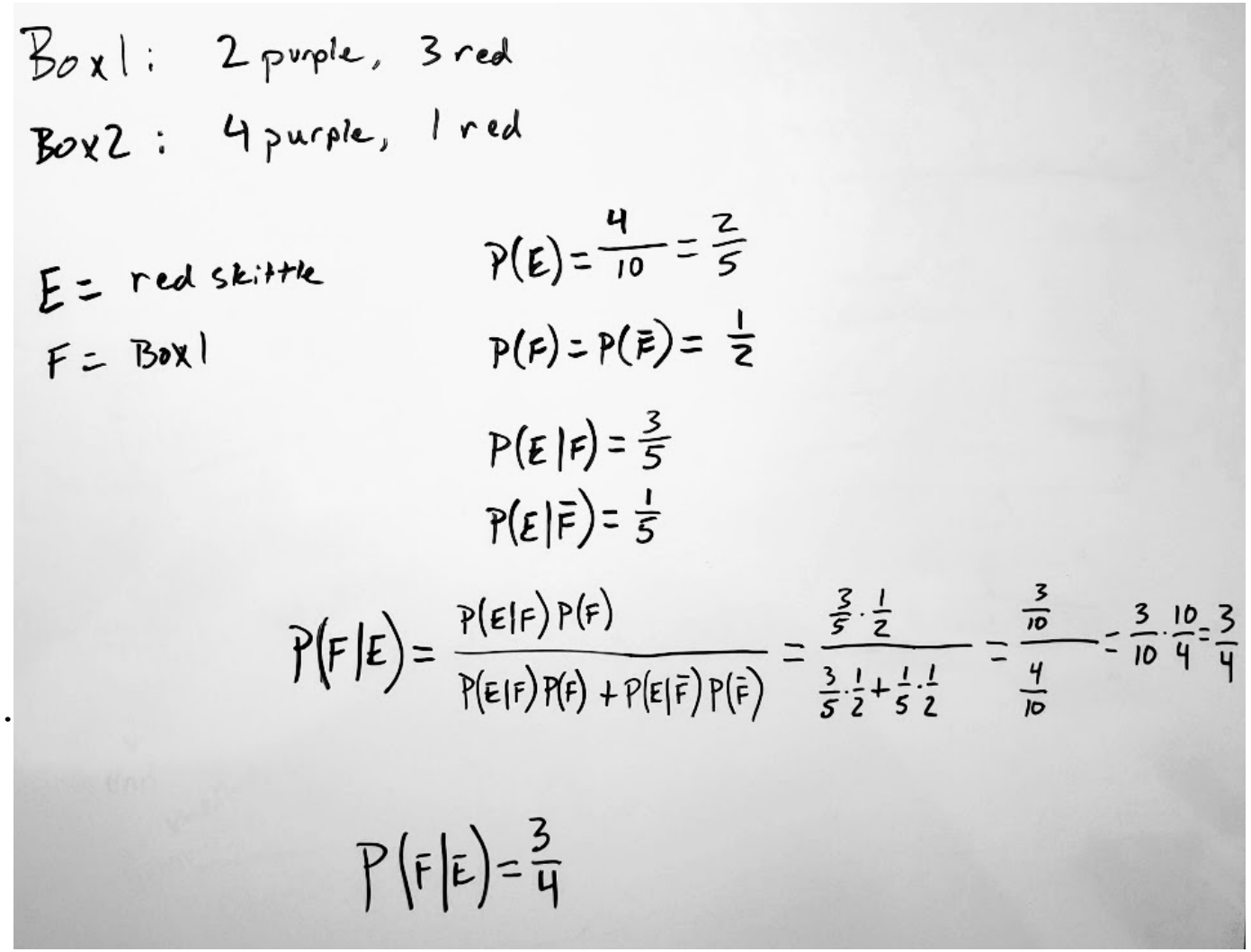
Suppose you draw a skittle from one of two boxes that you choose at random. The first box contains two purple skittles and three red skittles. The second box contains four purple skittles and one red skittle.

What is the probability you selected from the first box if you got a red skittle?

Let $E$ represent the event of drawing a red skittle. What is $P(E)$?

Let F represent the event of choosing from box 1. What is $P(F)$?

$P(E|F) = ?$ $P(E|\overline{F}) = ?$




We want to find $P(F|E)$. $P(F|E) = \frac{3}{4}$

# As a Class

Challenge 12.4.1

# With a partner

Additional Exercises

- 12.3.6
- 12.4.1
- 12.4.4