Sequences

Homework Review

Review Homework 07 with a partner.

Focus on:

7.4

7.6

7.7

Sequences

- Ordered list of elements
- Special **function** in which **domain** is a set of consecutive integers (e.g., 1, 2, 3, 4, ...)
- Many different sequences used in discrete mathematics
- We study them and try to find a way to express them

- Examples:

0, 1, 4, 9, 16, 25, 36, ...	a_n^2
0, 1, 8, 27, 64, 125, ...	a_n^3
1, 2, 4, 8, 16, 32, 64, ...	2^n
1, 1, 2, 6, 24, 120, 720, ...	$a_n!$
0, 1, 1, 2, 3, 5, 8, 13, ...	$a_n = a_{n-1} + a_{n-2},$ for $n > 1$ and $a_0 = 0, a_1 = 1$

Two special types of sequence

Arithmetic progression

1, 3, 5, 7, 9, ...

0, 7, 14, 21, ...

Geometric progression

1, 2, 4, 8, 16, 32, ...

1, 5, 25, 125, 625, ...

Arithmetic progression

A sequence of numbers separated by a fixed *common difference*.

$a_n = a_0 + n \cdot d, \text{for } n \geq 0$

Interval = 1: 1, 2, 3, 4, 5, ...

Interval = 2: 1, 3, 5, 7, 9, ...

Interval = 5: 1, 6, 11, 16, 21, ...

Interval = 5: 0, 5, 10, 15, 20, ...

How would you do this in Python?

```
range(start, stop, step)
```

Does this work if the common difference is not an integer?

Example

Geometric progression

A sequence of numbers separated a common ratio.

$a_n = a_0 \cdot r^n$ for $n \geq 0$

Multiple = 1: 1, 1, 1, 1, 1, ...

Multiple = 2: 3, 6, 12, 24, ...

Multiple = 2: 1, 2, 4, 8, 16, 32, ...

Multiple = 3: 1, 3, 9, 27, 81, ...

Multiple = 3: 4, 12, 36, 108, 324, ...

Multiple = 5: 0, 5, 25, 125, 625, ...

How would you do this in Python?

```
[a_0 * r**n for n in range(0, end)]
```

Example

Additional Exercises:

8.1.1 a, d, f, g (use python)

8.1.3 a-d (use python)

Recurrence Relations

Defines the next term in a sequence using earlier terms in the sequence.

Arithmetic sequence as a recurrence relation:

$a, a + d, a + 2d, a + 3d, \ldots, a + nd$

Each term looks like: $a + nd$

To define a recurrence relation, we rewrite to define each term in terms of the previous terms

$a_n = a_{n-1} + d, \text{for } n \geq 1$

And we need to add a base case to define the very first term:

$a_0 = a$

Recurrence Relations

Defines the next term in a sequence using earlier terms in the sequence.

Geometric sequence as a recurrence relation:

$a, ar, ar^2, ar^3, \ldots, ar^n$

Each term looks like: $a \cdot r$

To define a recurrence relation, we rewrite to define each term in terms of the previous terms

$a_n = a_{n-1} \cdot r, \text{for } n \geq 1$

And we need to add a base case to define the very first term:

$a_0 = a$

Python examples

[Recurrence relations in Python]()

Additional Exercises:

8.2.1 a-c (use python)