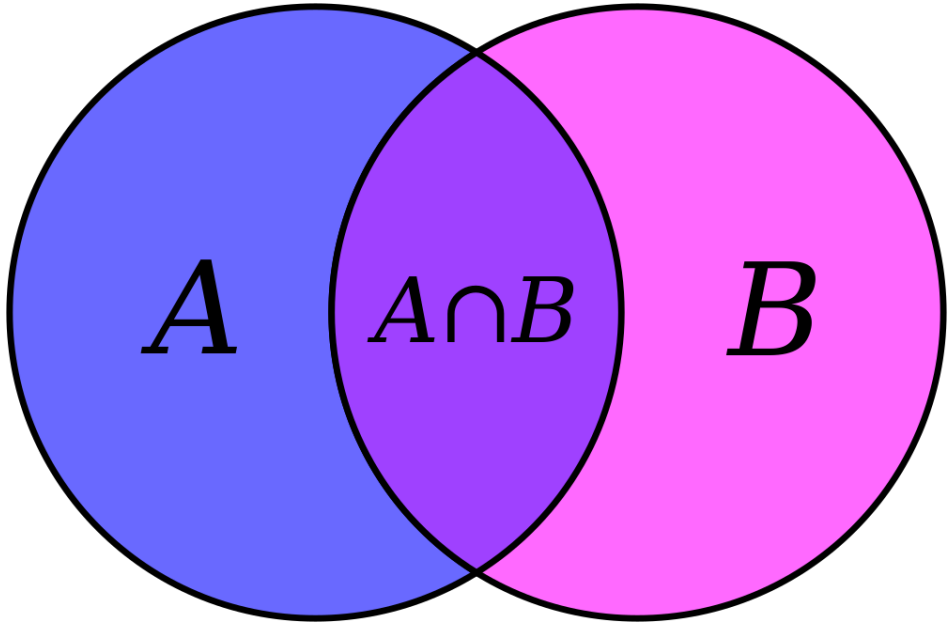
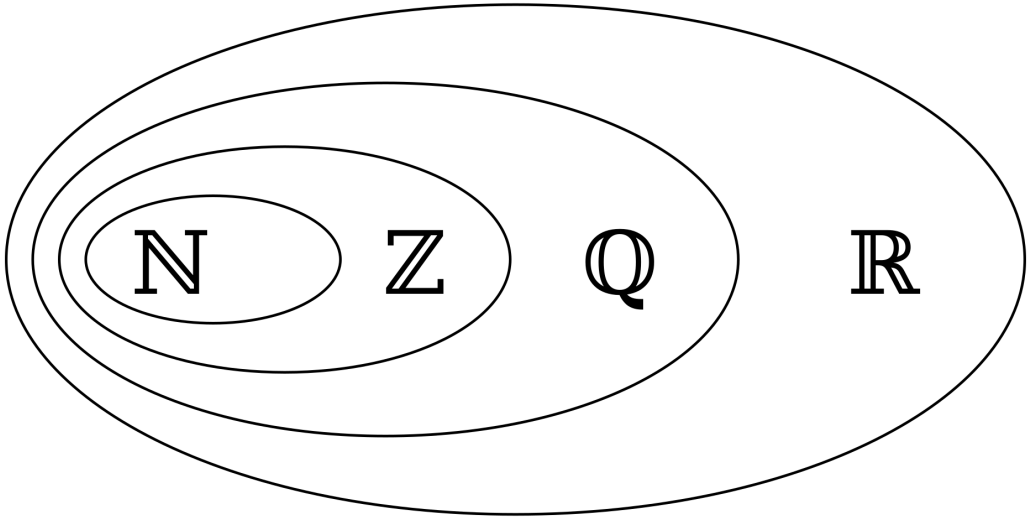
Sets

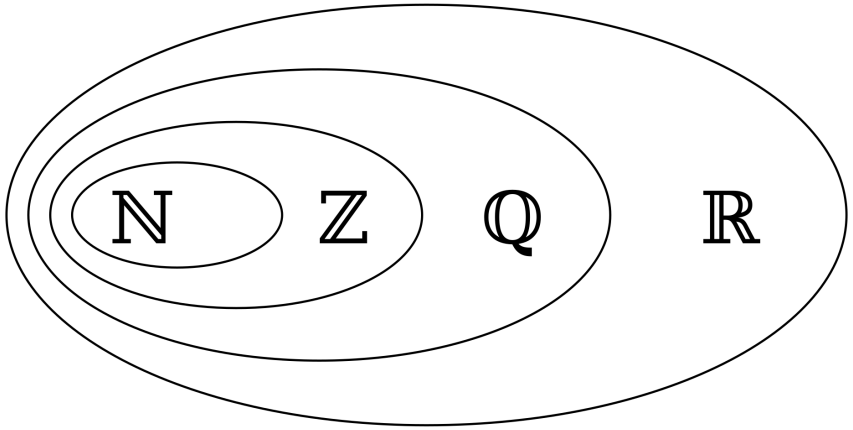




Review some symbols

Logic	Python	Example
$\in$	in	$x \in A$
$\notin$	not in	$x \notin A$
$\emptyset$	set( )	$\emptyset$
$\wedge$	and	$p \wedge q$
$\vee$	or	$p \vee q$
$\oplus$	^	$p \oplus q$
$\neg$	not	$\neg p$

Some special sets



$\mathbb{N}$	Natural Numbers	{1,2,3,4, … } (although some people include 0)
$\mathbb{Z}$	Integers	$\{\dots, -2, -1, 0, 1, 2, \dots\}$
$\mathbb{Q}$	Rational numbers	Numbers that can be represented as a fraction, such as $-\frac{3}{4}, \frac{1}{2}, \frac{5}{8}, \frac{13}{13}, \dots$
$\mathbb{R}$	Real numbers	All numbers that can be represented on a continuous number line

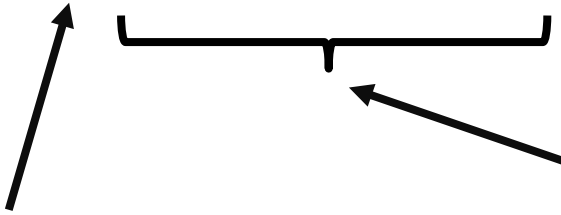
Set Builder Notation

Roster Notation – List all the members of a set. Works well for small sets.

$A = \{2,4,6,8,10\}$

Set Builder Notation – Create or "build" a set with some property or properties. Works to specify any size set.

$A = \{x \in \mathbb{Z}: 0 < x \leq 10 \text{ and } x \text{ is even}\}$



$\mathbb{Z}$ is the domain, or the set from which elements of the set A will be taken.

This part of the clause gives the conditions that specify membership in A.

Logical & Set Operators Compared

Logic		Set		Examples
Conjunction	$\wedge$	Intersection	$\cap$	$p \wedge q$, $A \cap B$
Disjunction	$\vee$	Union	$\cup$	$p \vee q$, $A \cup B$
Negation	$\neg$	Complement	$\bar{}$	$\neg p$, $\bar{A}$

Subsets and Supersets

$B \subset A$	B is a proper subset of A	

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$B \subseteq A$	B is a subset of A	"B is *less than or equal to* A" $\leq$
$A \supset B$	A is a proper superset of B	"A is *greater than* B" $>$
$A \supseteq B$	A is a superset of B	"A is *greater than or equal to* B" $\geq$

Powerset

The set of all subsets of a set

$A = \{a, b, c\}$

What is the powerset of A, which is denoted as $P(A)$?

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

What is the cardinality of A?

$2^3 = 8$ Why?

Some python examples

Cartesian Product

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

Example:

$A = \{1, 2\}$

$B = \{a, b, c\}$

What is $A \times B$? $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

What is $B \times A$? $B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$

Note: $A \times B \neq B \times A$

Find the Cartesian Product of A with itself where A = {0, 1}:

$A \times A = ?$

{(0,0), (0,1), (1,0), (1,1)}

In Python:

```
A = [0,1]
[(x,y) for x in A for y in A]
```

Can you generate $\{0,1\}^5$ and list them all?

Example

Example:

Given the sets $A = \{a, b, c\}$ and $B = \{1,2,3\}$, what is the Cartesian Product $A \times B$?

$$\{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3),(c,1),(c,2),(c,3)\}$$

How many elements are in the Cartesian Product?

$$3 \text{ x } 3 = 9$$

In general, the size of the Cartesian Product is the size of A times the size of B

$$|A \times B| = |A| \cdot |B|$$

You own an ice cream shop. If you offer 15 different flavors of ice cream, how many types of 3-scoop cones might you be able to claim in your advertising if you use the Cartesian product to build the set of possibilities?

Write Python code! Can you list all the possibilities easily?

Answer: 3375

Example

Bit strings

$\{0,1\}^n$ represents the set of all bitstrings of length n.

For example:

$\{0,1\}^2 = \{00, 01, 10, 11\}$

$\{0,1\}^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

What is the cardinality of $\{0,1\}^8$?

Partitions

A partition of a set is A is a set of one or more nonempty subsets of A: $A_1, A_2, A_3, \ldots$, such that every element of A is in exactly one set.

In symbolic terms,

1. $A_1 \cup A_2 \cup A_3 \cup \cdots = A$

2. If $i \neq j$ then $A_i \cap A_j = \emptyset$

Example:

Let $A = \{a, b, c, d\}$. Some of the partitions of A are:

- $\{\{a\}, \{b\}, \{c, d\}\}$
- $\{\{a, b\}, \{c, d\}\}$
- $\{\{a\}, \{b\}, \{c\}, \{d\}\}$

There are 15 different partitions of a set with four elements. For more, look up the *Bell numbers*.

Partitions

What are some ways to partition the set $S = \{1,2,3,4,5,6,7,8,9,10\}$? List at least three.

$\{\{1, 3, 5, 7, 9\}, \{2, 4, 6, 8, 10\}\}$	Evens and odds
$\{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{10\}\}$	Number of digits
$\{\{1, 4, 7, 10\}, \{2, 5, 8\}, \{3, 6, 9\}\}$	Modulo 3
$\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}\}$	All separate