

CSE 280

Discrete Mathematics



Syllabus / Course Layout

Schedule (reading, homework)

Reading / Activities – ZyBooks

Homework – Google Colab, grading, how to submit

Groups – 2-4, 3 is ideal

Grading

Resubmission policy

Questions?

What is Discrete Mathematics?

"the branch of mathematics dealing with objects that can assume only distinct, separated values" – Wolfram Mathworld

In contrast with continuous mathematics, which deals with things that vary smoothly / continuous numbers (e.g., calculus)

Topics

Propositions and Logic

Predicates and Quantifiers

Sets

Functions

Relations

Number Theory
(Integers, prime numbers)

Combinatorics

Sequences and Summations

Probability

Graphs

Trees

$$p \wedge q \quad p \vee q$$

$$\forall x P(x) \quad \exists x P(x)$$

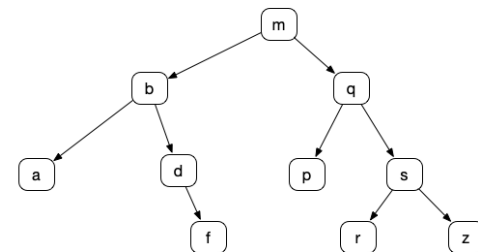
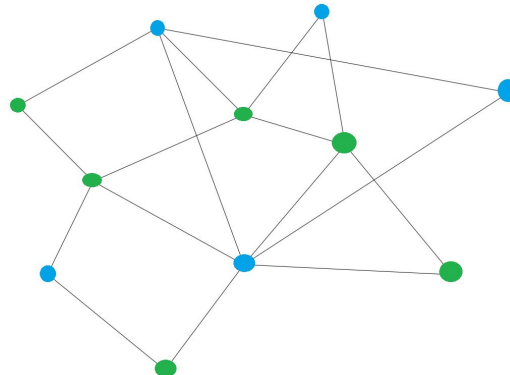
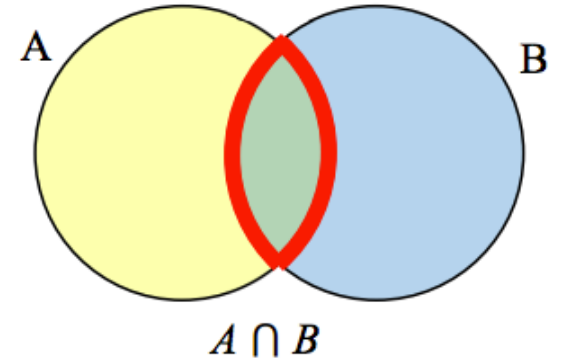
$$F(x) = x^2, x \in \mathbb{R}$$

$$R = \{(a, b) \mid a + b = 6\}$$

$$2, 3, 5, 7, 11, 13, 17, 19, \dots$$

$$\binom{11}{3}$$

$$\sum_{i=1}^4 i = 1 + 2 + 3 + 4 = 10$$



Google Colab Demo

Saving your work

Markdown / LaTeX

Python

Propositions & Logic

$$p \wedge q \rightarrow r$$

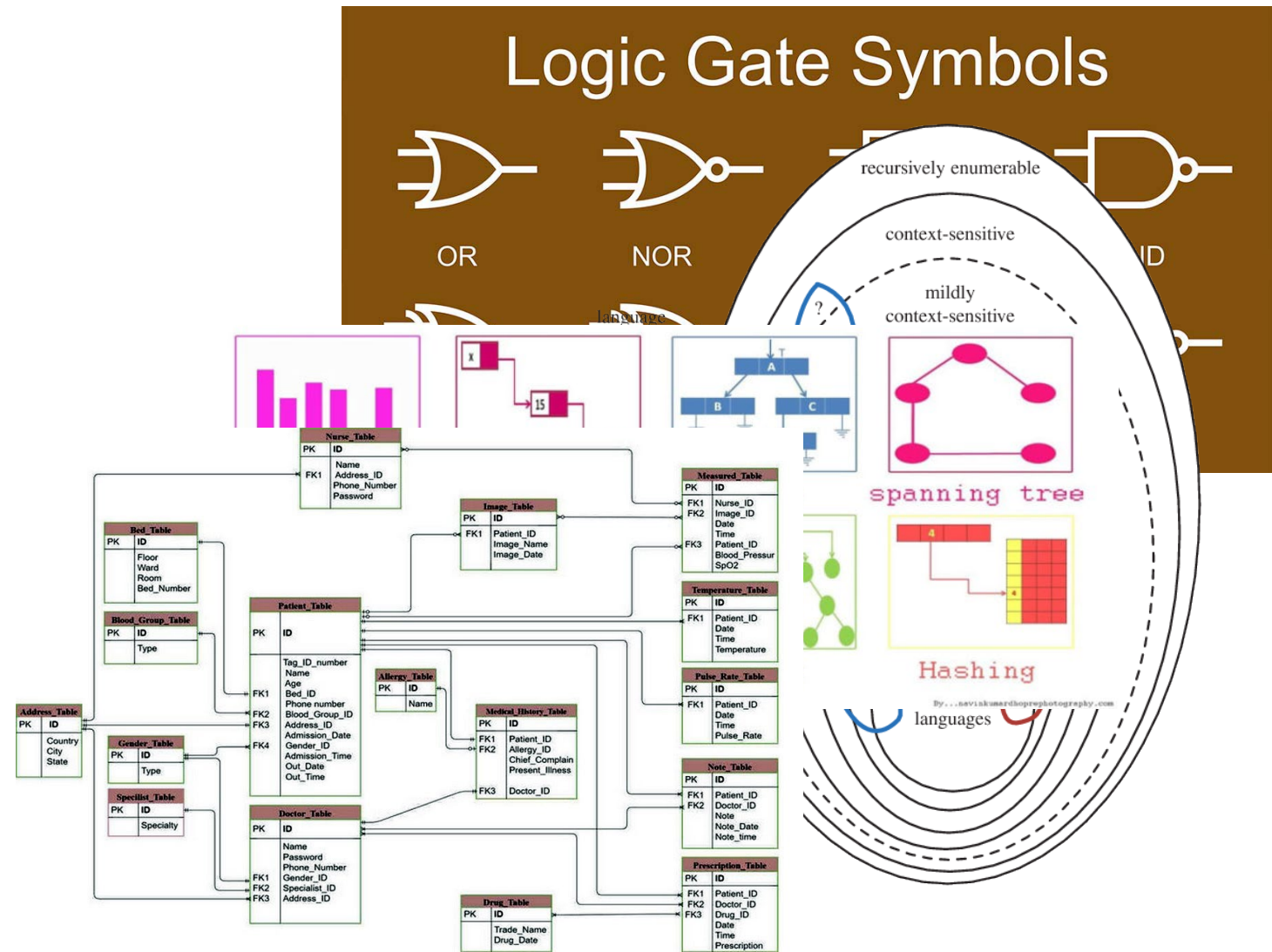
$$\neg p \vee \neg q$$

$$\neg(p \wedge r) \vee (q \wedge s)$$

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$$

Why do I need this?

- Rules of Logic are everywhere:
 - Mathematical reasoning
 - Computer circuits (Logic gates)
 - Formal Languages (computer languages)
 - Data Structures
 - Algorithm Design
 - Compiler Design
 - Relational Database Theory
 - Computability Theory
 - ...
- All built on a foundation of formal logic



Why do I need this?

- Why is it important to think logically?
 - Logic sharpens your thinking, allowing it to “cut” more powerfully.
- Why know logic (formal or informal)?
 - Logic gives you a means to mechanize reasoning.
 - Mechanizing reasoning means programming a computer to do it.

Propositions

A proposition is...

A declarative sentence.

In other words, a sentence that is either *True* or *False*

The sky is blue

Today is Tuesday

We are in a discrete math class

$2 + 2 = 5$

Propositions

We represent propositions using a variable, such as p , q , r , s

Examples:

p = The sky is blue

q = It is Tuesday

r = My dog's name is Eevee



Propositions

We can build compound propositions by following these rules:

A proposition is...

A <i>variable</i> representing a <i>declarative sentence</i>	p
<i>proposition</i> preceded by <i>not</i>	$\neg p$
<i>proposition</i> <i>connective</i> <i>proposition</i>	$p \wedge q$ $p \vee q$ $p \oplus q$

p = We are in a discrete math class

q = It is Tuesday

$p \wedge q$ = We are in a discrete math class *and* it is Tuesday.

Connective	Name	Meaning
\wedge	conjunction	and
\vee	disjunction	inclusive or "either or"
\oplus	exclusive or	exclusive or "one or the other but not both"

Translating to/from English

p = It is raining

q = It is sunny

r = We will go outside

Translate the following into English:

$p \vee q$

It is raining or it is sunny.

$(p \wedge \neg r) \vee (q \wedge r)$

It is raining and we will not go outside, or it is sunny and we will go outside.

$p \wedge q \wedge r$

It is raining and it is sunny and we will go outside.

Logic symbols in Python

Name	Logic	Example	Python	Python Example
conjunction	\wedge	$p \wedge q$	and	p and q
disjunction	\vee	$p \vee q$	or	p or q
exclusive or	\oplus	$p \oplus q$	^	p ^ q
negation	\neg	$\neg p$	not	not p

Examples

Truth Tables

How many rows will be in the truth table for the following propositions?

$p \wedge q$ 4 rows

$p \vee q \wedge r$ 8 rows

$(p \wedge \neg q) \vee (r \wedge q)$ 8 rows

$p \wedge q \wedge r \wedge s$ 16 rows!

There are 2^n rows in a truth table with n variables

Truth Tables

Let's create the following truth table.

$$(p \wedge \neg q) \vee (r \wedge q)$$

Practical Uses of Logical Operators

- [Search Engines](#)
- [Conditionals in programming](#)
- Can you think of any others?

Group Activity

With a group of 2-3 people, work through the following Additional exercises at the bottom of reading 1.2

- 1.2.1

Conditionals or Implications

$$p \rightarrow q$$

p = You pass the final exam

q = You will get an A in the class

Conditional statement: If you pass the final exam, then you will get an A in the class.

(If you don't pass the final exam, then we don't know if you'll get an A or not. It is not implied by this statement)

Conditionals or Implications

$$p \rightarrow q$$

p : You are in this class
 q : You are a BYU-I student

if p , then q

if p , q

p only if q

p implies q

p is sufficient for q

q if p

q whenever p

q is necessary for p

You being in this class is sufficient *for me to conclude that* you are a BYU-I student.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

[Python example](#)

Conditionals: $p \rightarrow q$

$$p \rightarrow q$$

if p, then q

if p, q

p only if q

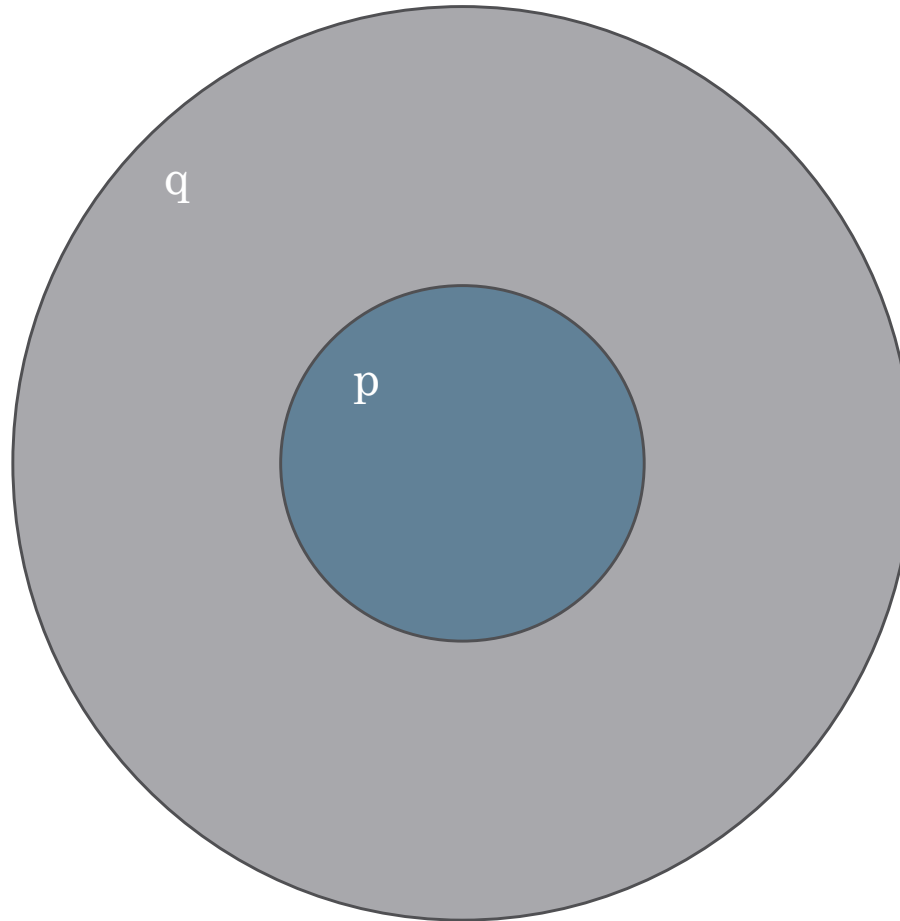
p implies q

p is sufficient for q

q if p

q whenever p

q is necessary for p



p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditionals: $p \rightarrow q$

$p =$ *It is sunny*

$q =$ *We will go outside*

Conditional: $p \rightarrow q$

If it is sunny, then we'll go outside.

Converse: $q \rightarrow p$

If we go outside, then it is sunny.

Inverse: $\neg p \rightarrow \neg q$

If it is not sunny, then we will not go outside.

Contrapositive: $\neg q \rightarrow \neg p$

If we do not go outside, then it is not sunny.

Conditionals: $p \rightarrow q$

$p =$ *You pass the exam*

$q =$ *You get an A in the class*

Conditional: $p \rightarrow q$

If you pass the exam, then you get an A in the class.

Converse: $q \rightarrow p$

If you get an A in the class, then you passed the exam.

Inverse: $\neg p \rightarrow \neg q$

If you didn't pass the exam, then you won't get an A in the class.

Contrapositive: $\neg q \rightarrow \neg p$

If you don't get an A in the class, then you didn't pass the exam.

Biconditional

$$p \leftrightarrow q$$

p : You get an A in the class

q : You pass the final exam

You will get an A in the class if and only if you pass the final exam.

p if and only if q

p iff q

p is necessary and sufficient for q

if p then q and if q then p

if p then q and conversely

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Group Activity

With a group of 2-3 people, work through the following Additional exercises at the bottom of reading 1.3

- 1.3.1